Lovász Local Lemma as Double Counting

nonrepetitive sequences

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(based on joint work with Piotr Micek)

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It would be hopeless to replace the applications of many of the tools appearing in this book, including, for example, the second moment method, the Lovász Local Lemma and the concentration via martingales by counting arguments, even when these are applied to finite probability spaces.

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Outline

- Lovász Local Lemma (LLL)
- application to nonrepetitive sequences
- constructive proof of LLL (by Moser and Tardos)
- application of double counting to nonrepetitive sequences

Lovász Local Lemma (Lovász, Erdös 1975)

Let $\mathcal{A}$ be a finite set of events in a probability space. For $A \in \mathcal{A}$ let $\Gamma(A)$ be a subset of $\mathcal{A}$ satisfying that A is independent from the collection of events $\mathcal{A} - (\{A\} \cup \Gamma(A))$. If there exists an assignment of reals $x : \mathcal{A} \to (0, 1)$ such that

$$\textit{for all } A \in \mathcal{A} \ : \ Pr(A) \leq x(A) \prod_{B \in \Gamma(A)} (1 - x(B)),$$

then the probability of avoiding all events in $\mathcal{A}$ is at least

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Nonrepetitive sequences

A repetition of size h in a sequence is a substring of the form:

$$xx = x_1 \dots x_h x_1 \dots x_h.$$

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A sequence is nonrepetitive if it does not contain a repetition of any size.

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Question

How many symbols do we need to build arbitrarily long nonrepetitive sequence?

Nonrepetitive sequences - LLL

Fix $n \in \mathbb{N}$. Consider random word $\omega = \omega_1 \dots \omega_n$. (each letter chosen independently from the set of k symbols with uniform distribution)

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$$Pr(A_{i,m}) = k^{-m}$$

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$$x(A_{i,m}) := \frac{1}{6^m + 1}$$

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Result

For $k \geq 14$ the assumptions of LLL hold. Therefore with positive probability the word ω is nonrepetitive.

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There exist arbitrarily long nonrepetitive sequences over 3 symbols.

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Thue's substitution:

$$a \to abcab, \quad b \to acabcb, \quad c \to acbcacb$$

sequence chosen from lists

$L_1, \ldots, L_n$ – sets of symbols (called lists)

L_1	L_2	L_3	L_4	L_5	L_6	L_7
a	b	a	c	f	b	c
b	d	b	h	g	c	e
c	e	g			h	g
d	g	h				h
e		i				

A sequence $s_1, \ldots, s_n$ is chosen from $L_1, \ldots, L_n$ if $s_i \in L_i$ for all i.

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64 Alon, Grytczuk, Hałuszczak, Riordan 2002

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special case of the result on nonrepetitive colorings

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using the lefthanded local lemma by Pegden (2010)

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Answer: 3 or 4.

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Algorithm

for $i := 1$ **to** m
    $v_i :=$ random evaluation
while some $A \in \mathcal{A}$ holds
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Theorem (Moser, Tardos, 2010)

The expected number of resamples after which the algorithm finds an instance avoiding all events from $\mathcal{A}$ is at most

$$\sum_{A \in \mathcal{A}} \frac{x(A)}{1 - x(A)}$$

Nonrepetitive sequences

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Algorithm (Moser Tardos style)

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while $i \leqslant n$ **do**
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    there is exactly one repetition, say of size h
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Example of first few iterations (alphabet $\{a, b, c\}$)

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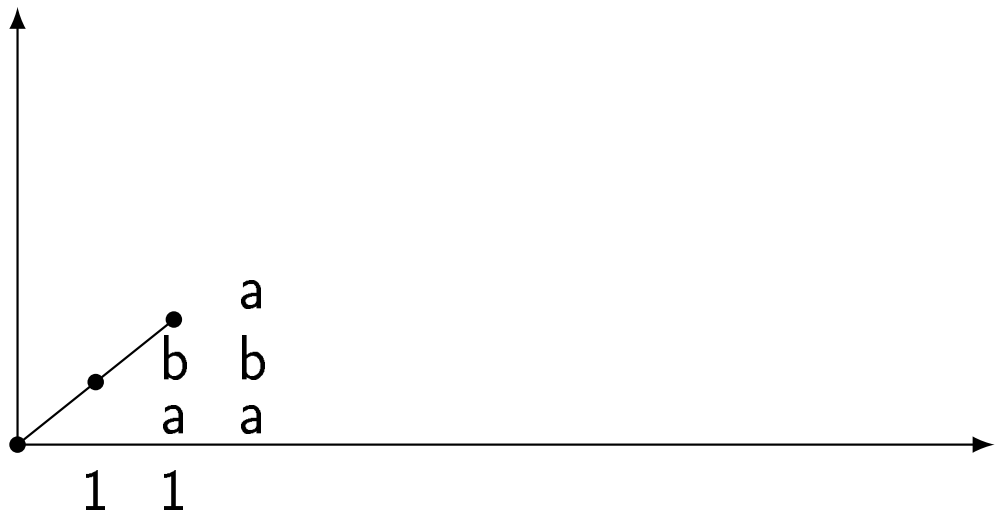

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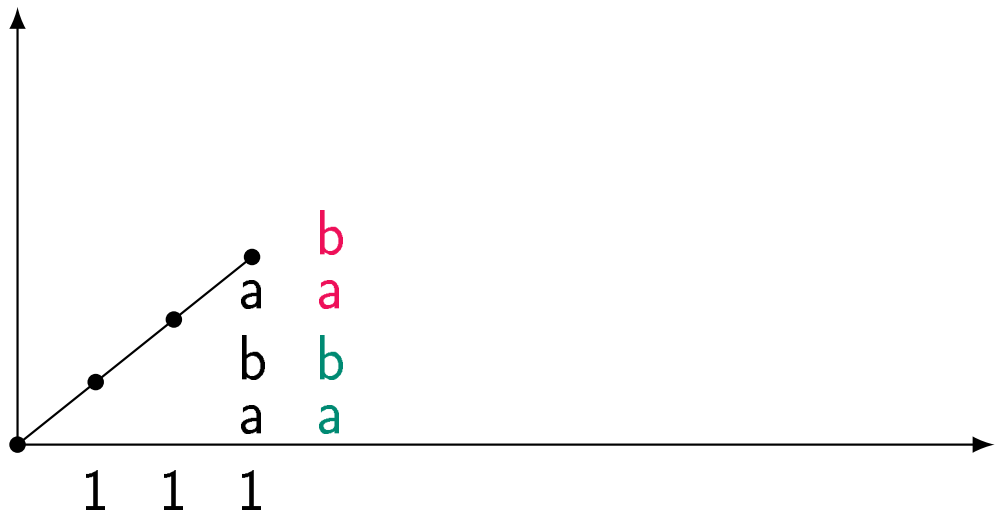

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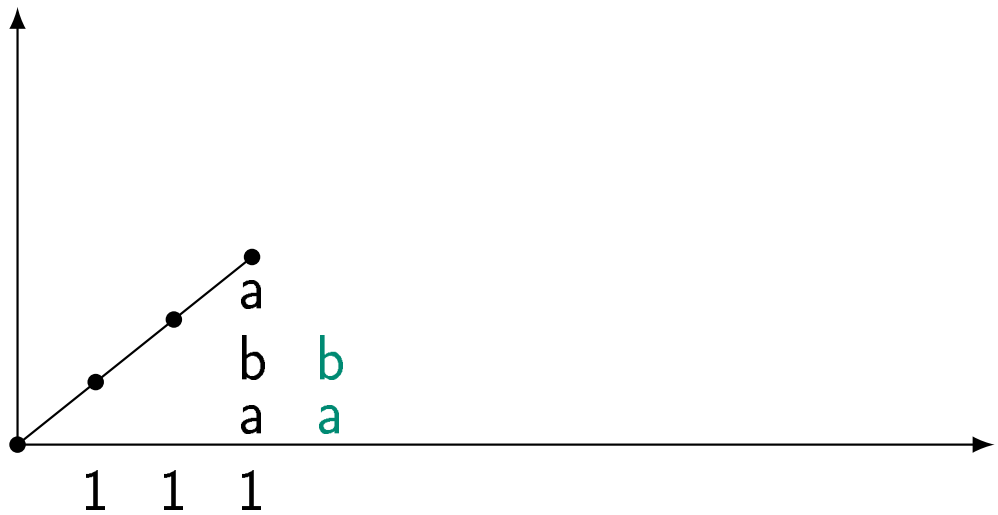

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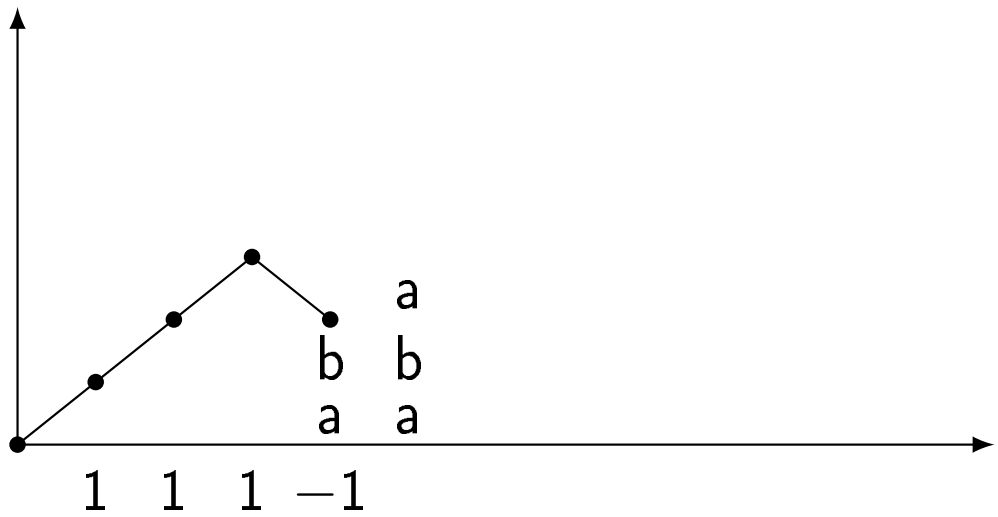

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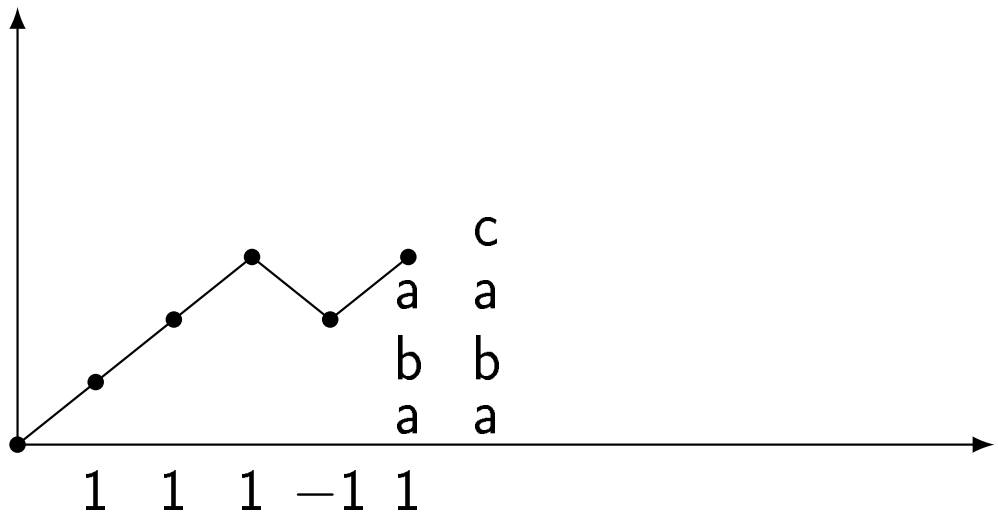

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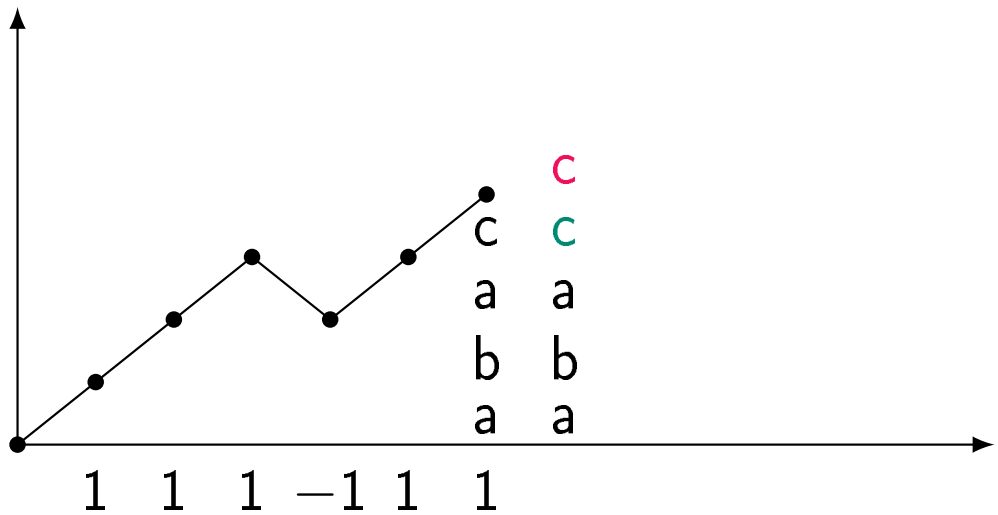

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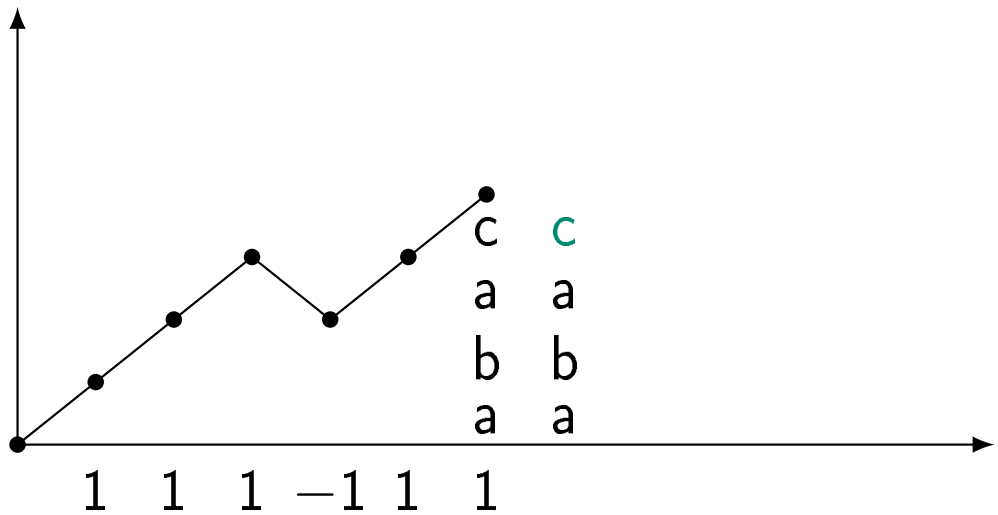

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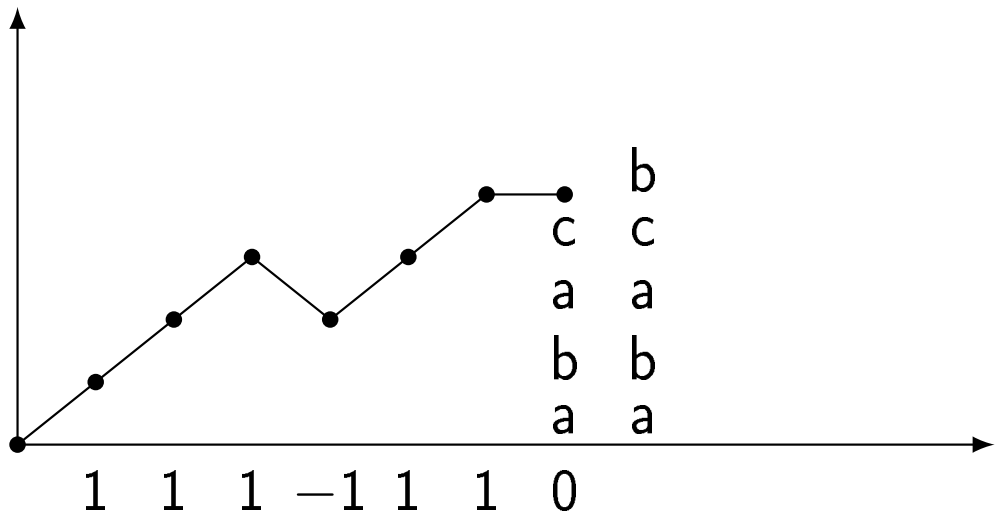

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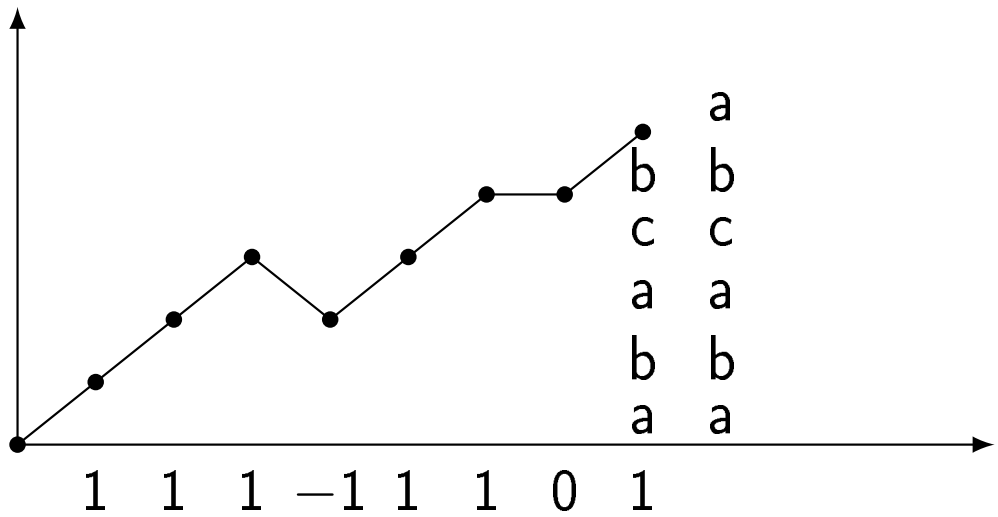

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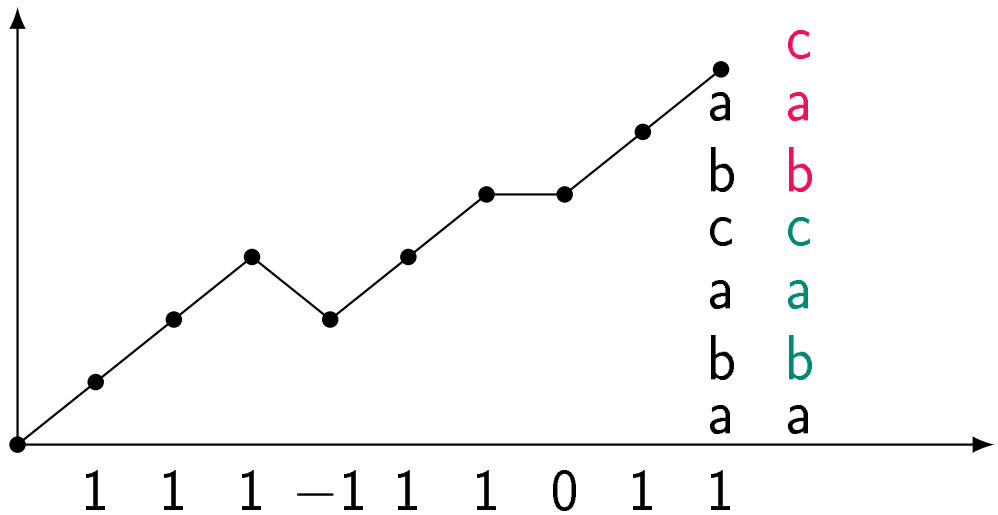

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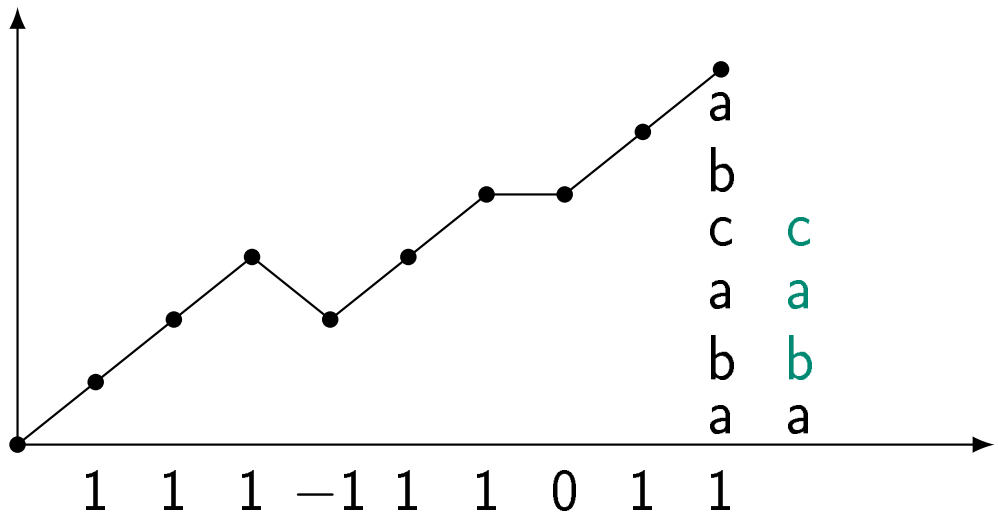

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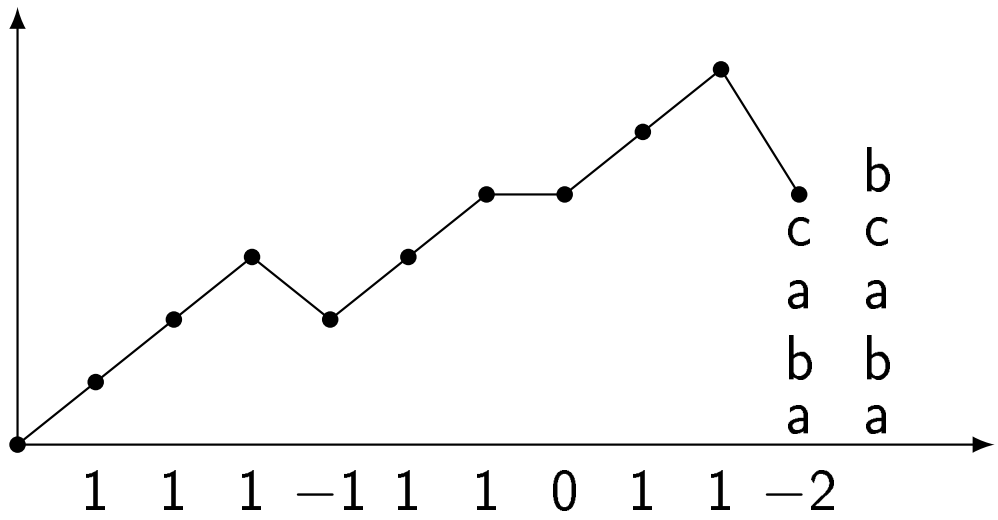

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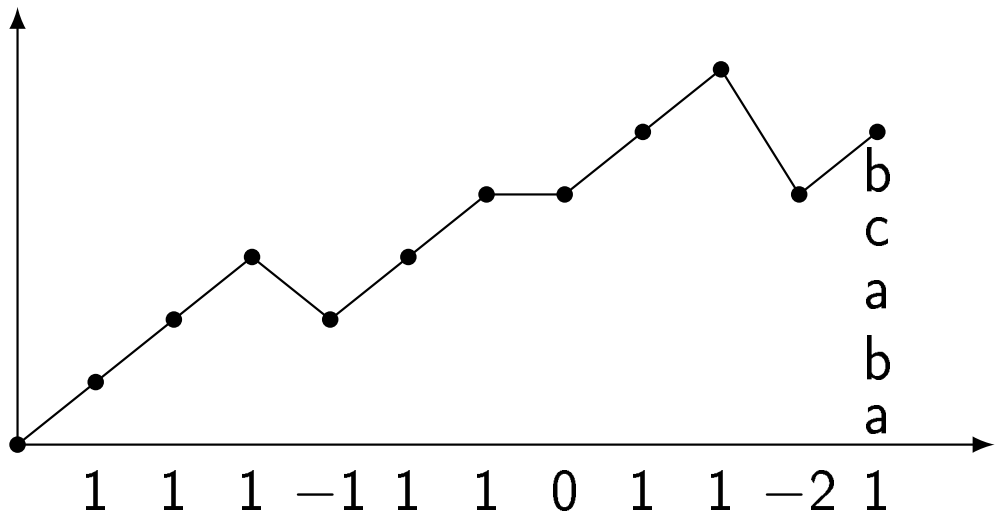

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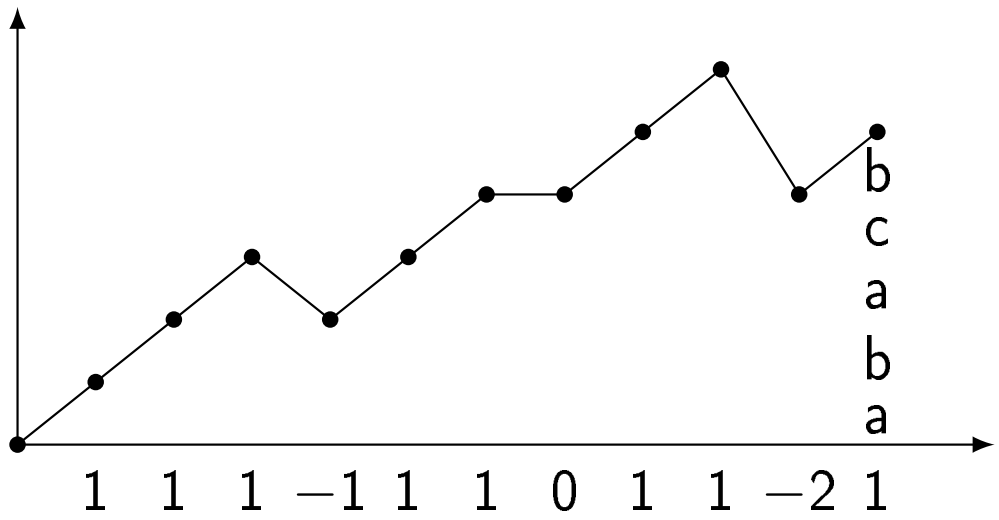

log

- $(1, 1, 1, -1, 1, 1, 0, 1, 1, -2, 1)$ – sequence of differences
- abacb – final word

logs vs random strings

Observation

Log encodes sequence of results of random experiments $r_1, \ldots, r_n$ in a lossless fashion.

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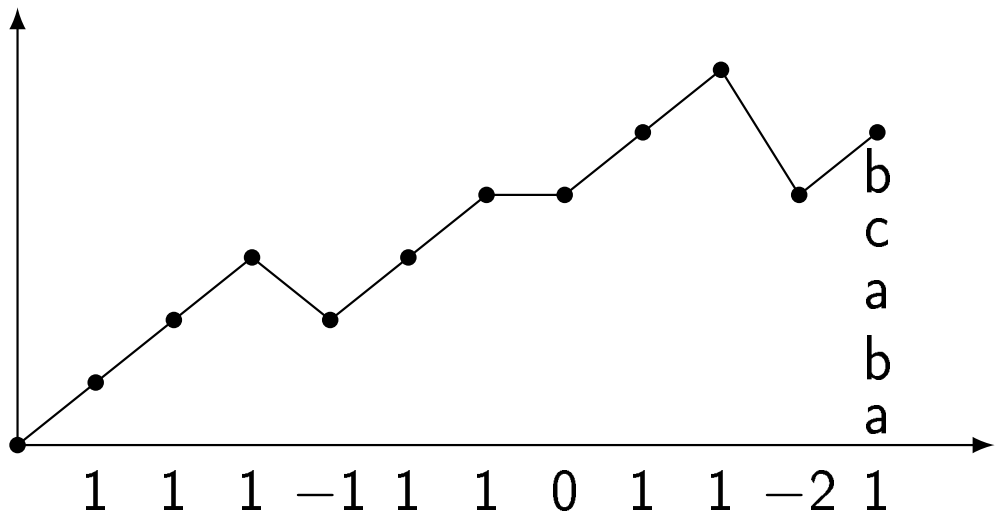

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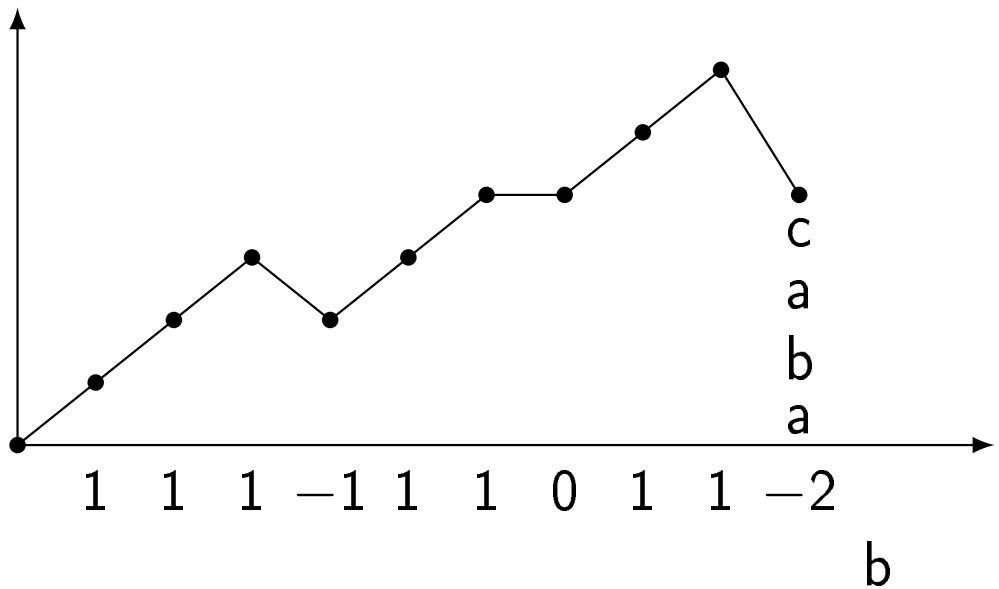

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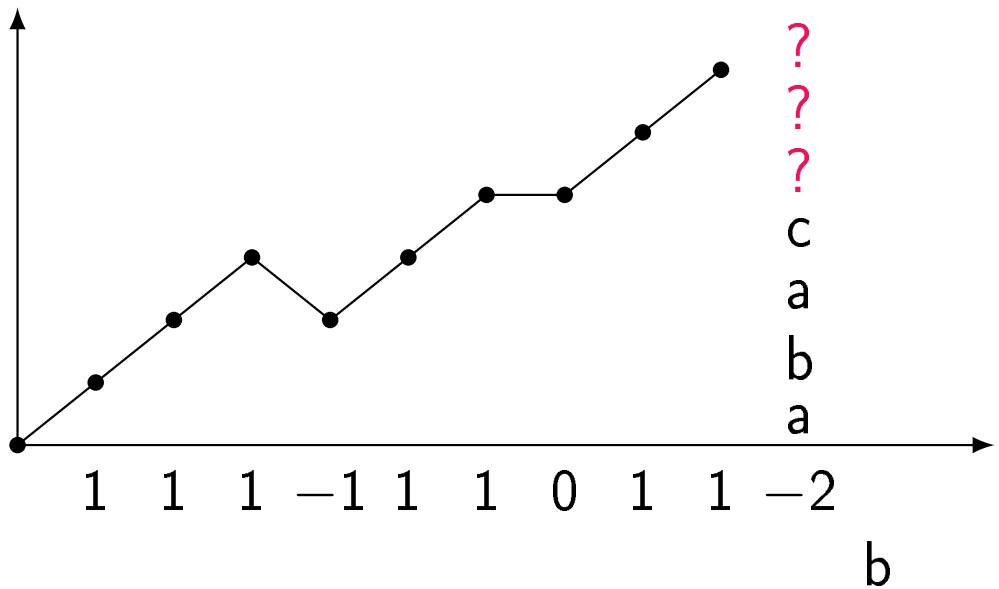

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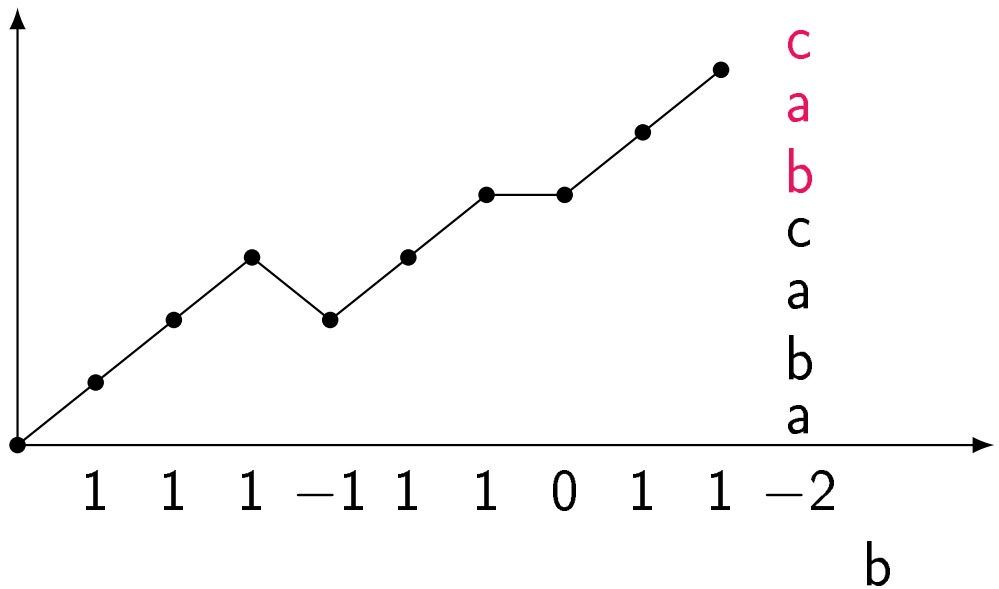

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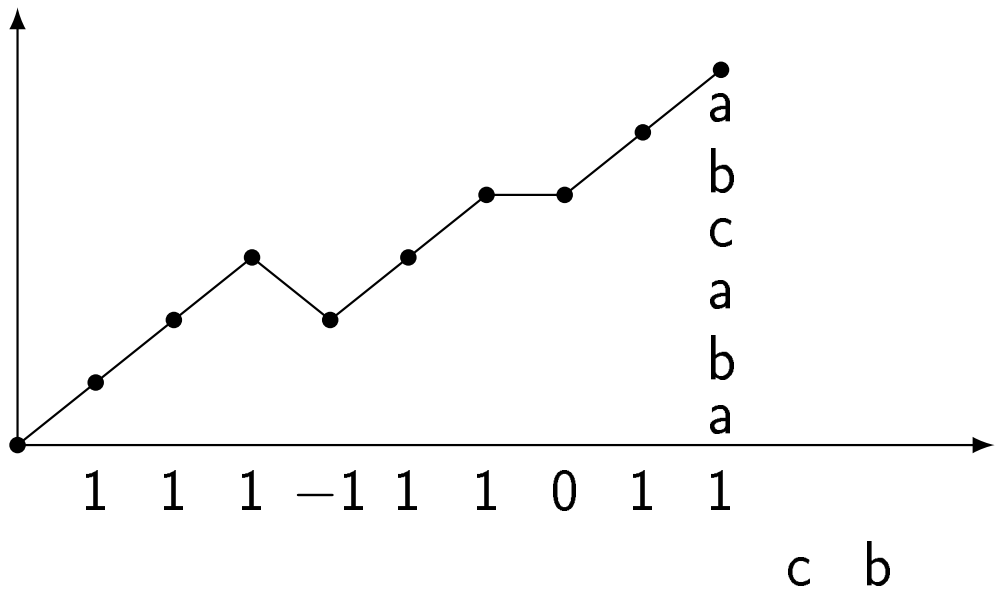

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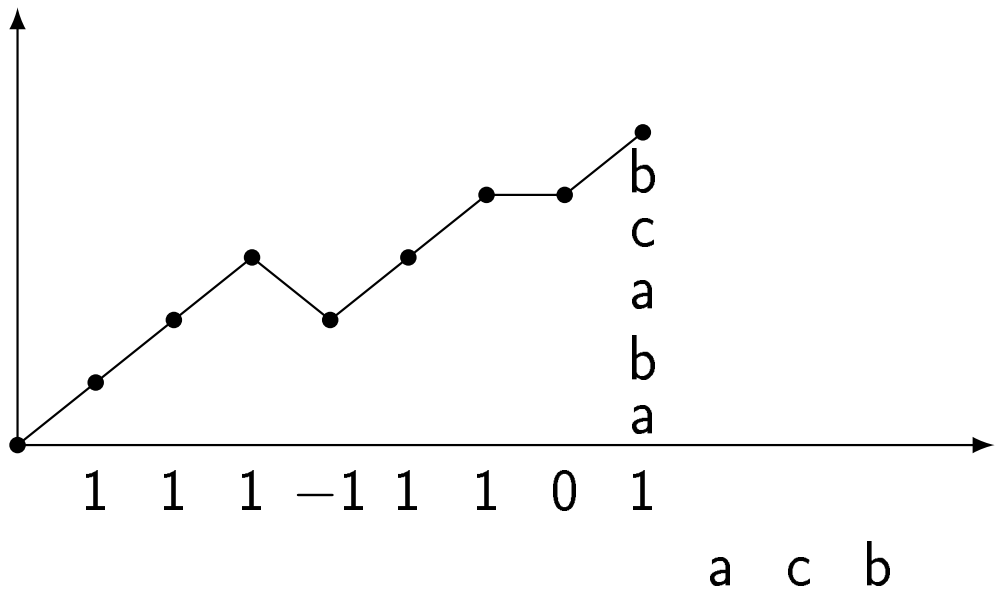

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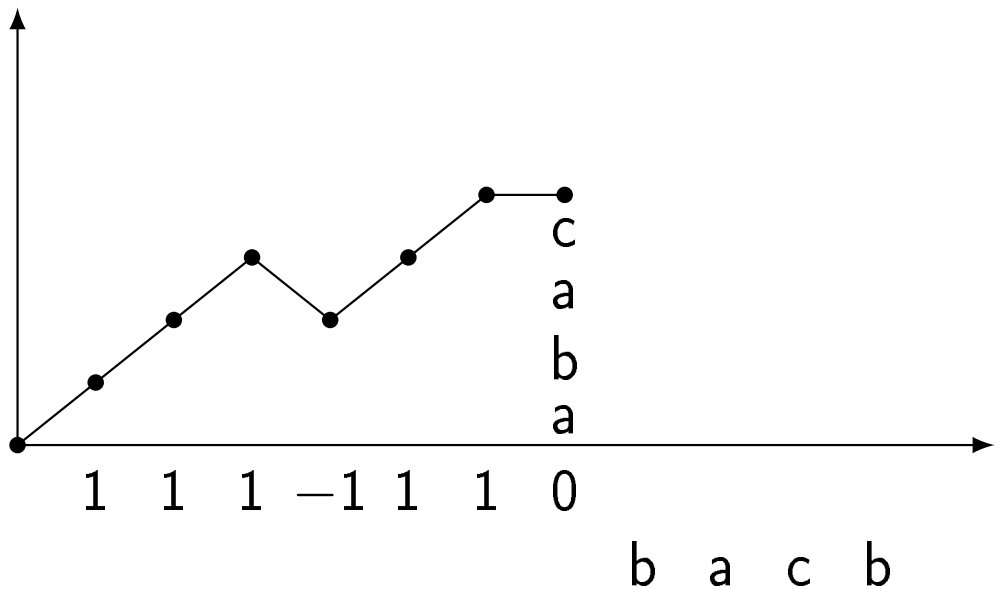

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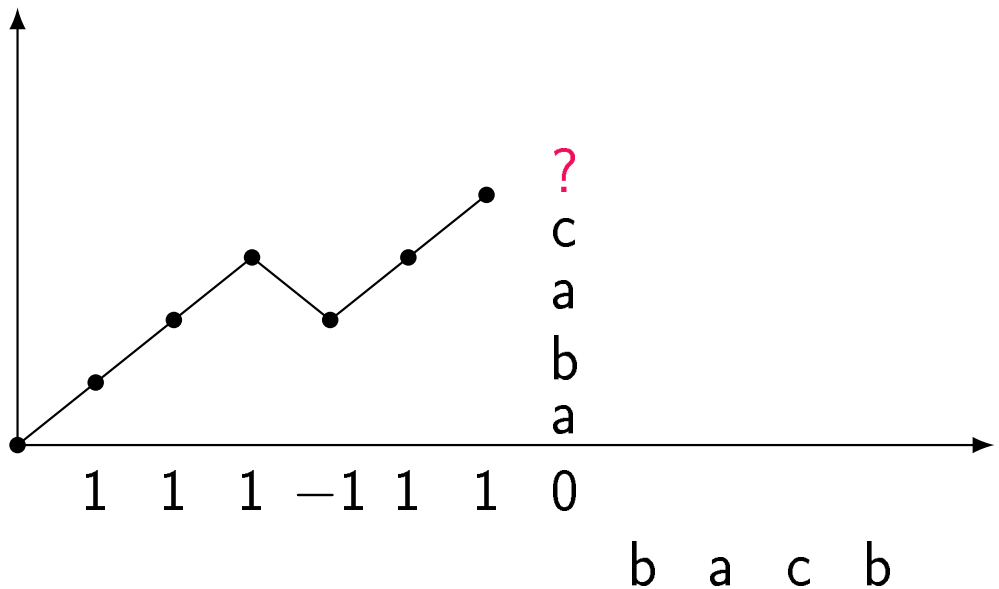

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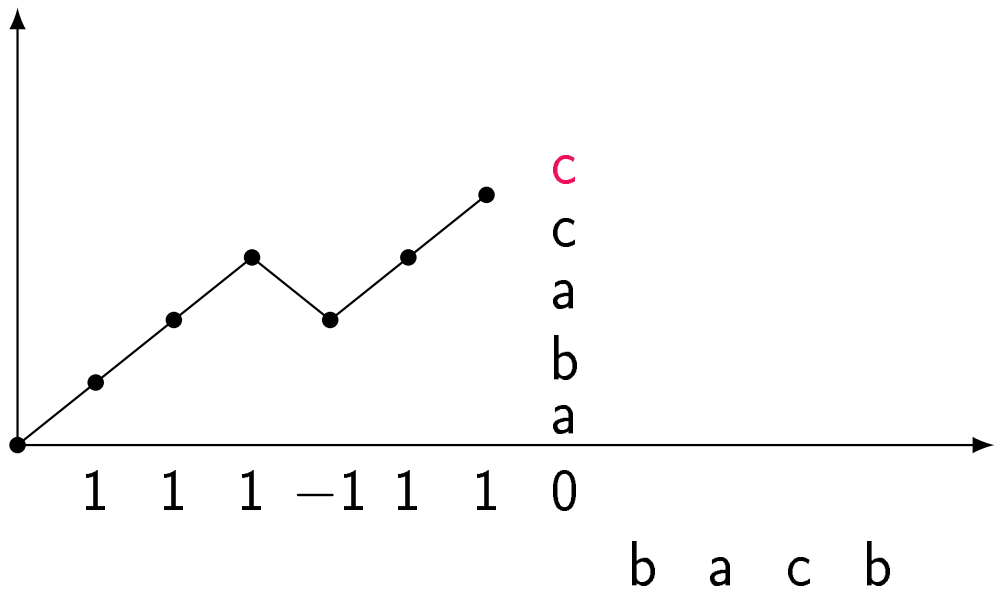

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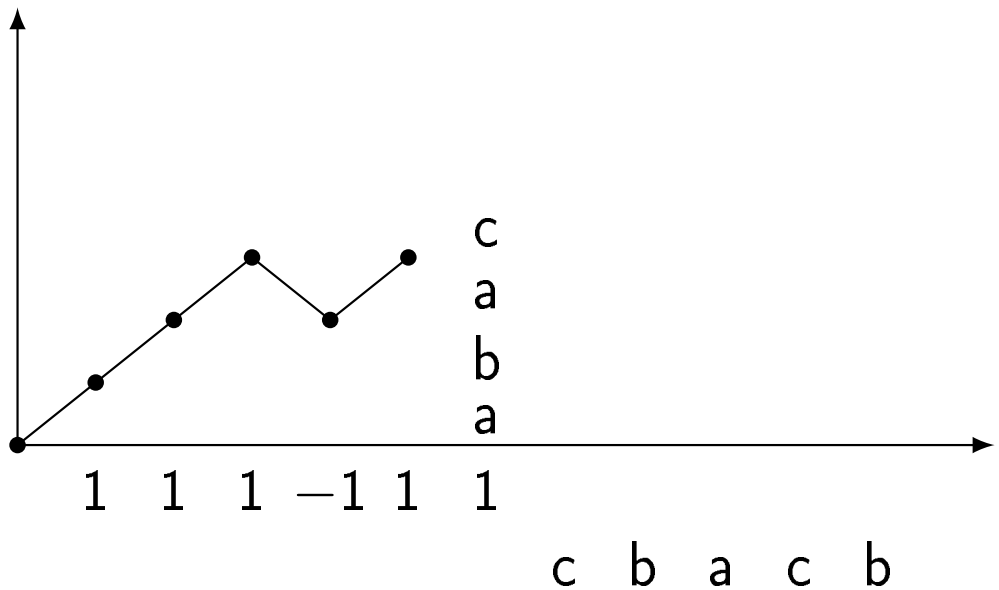

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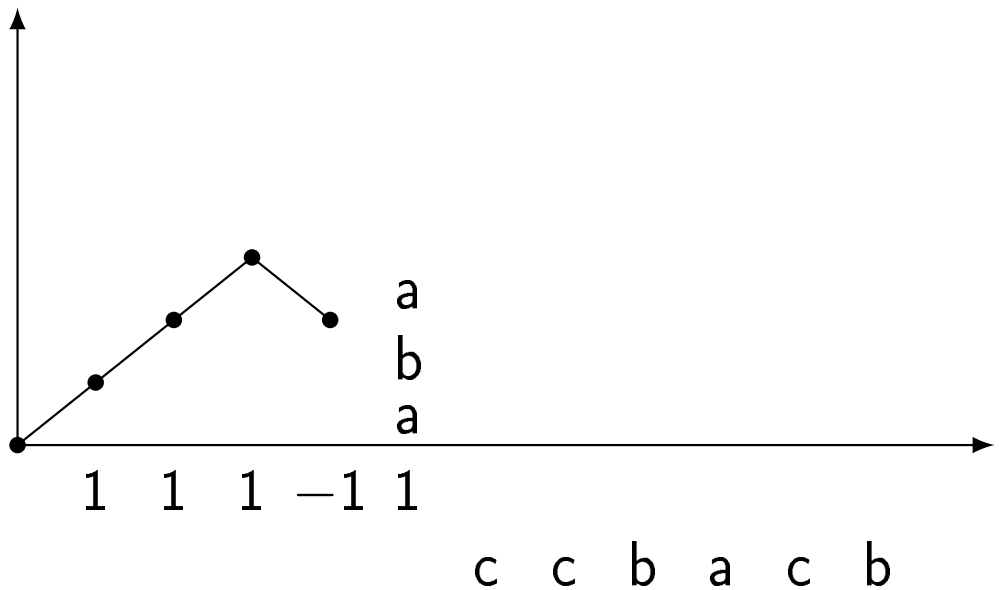

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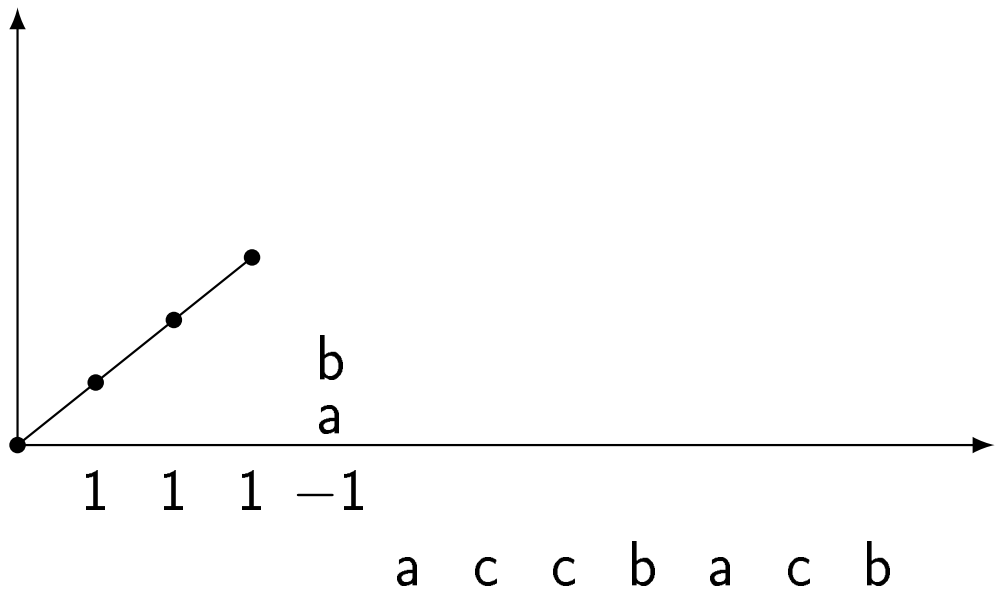

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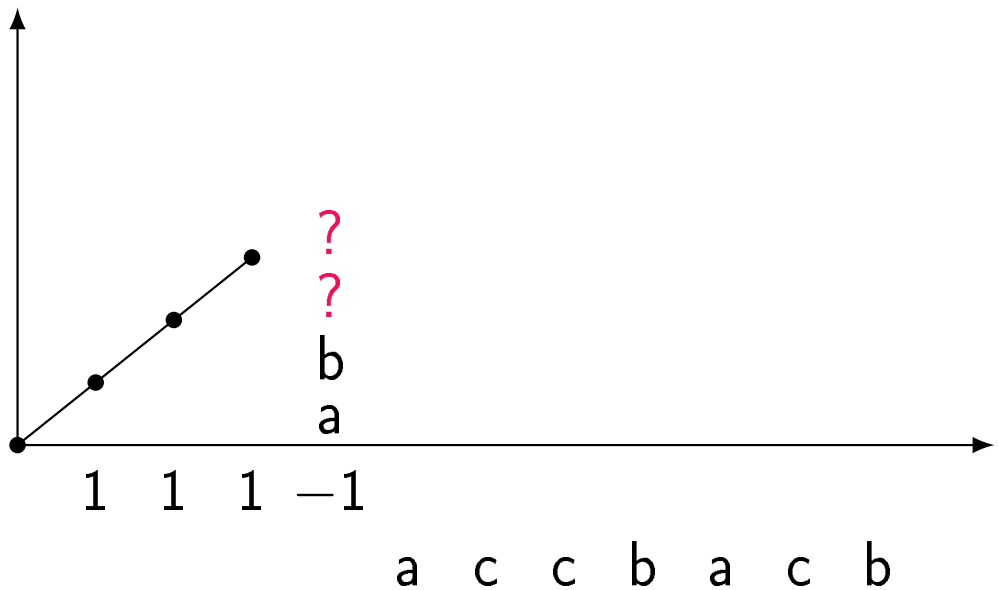

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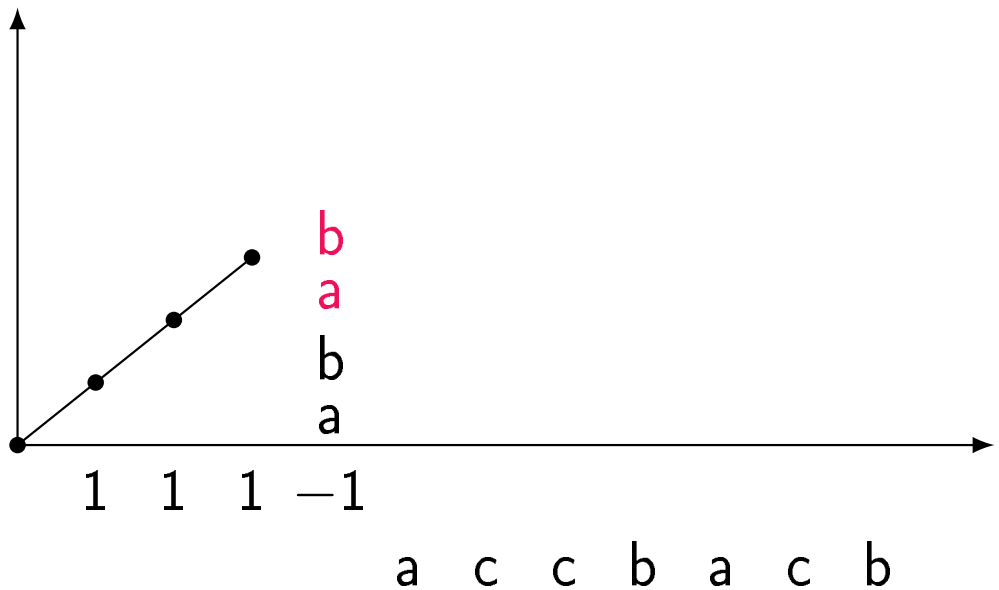

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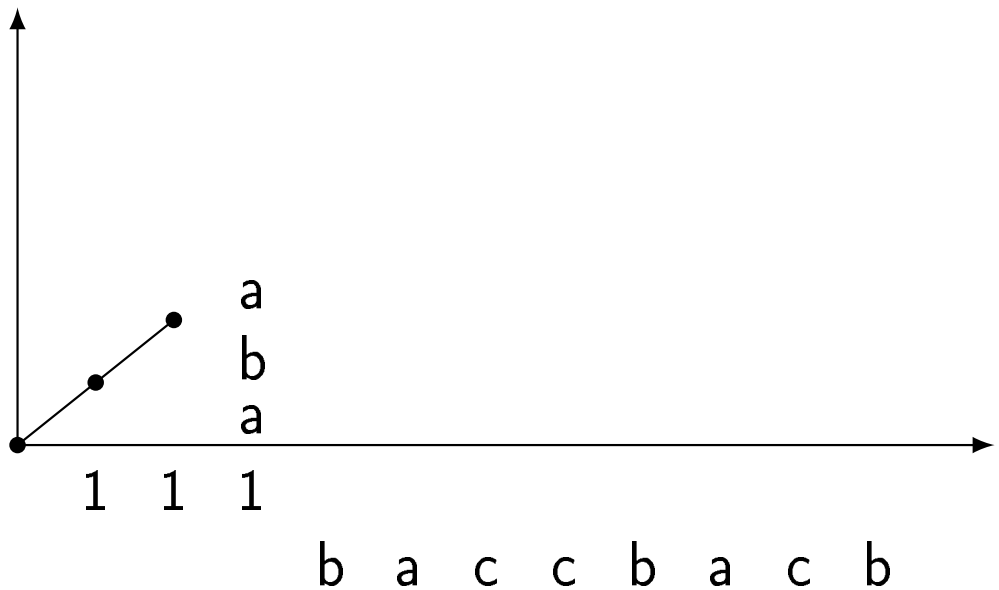

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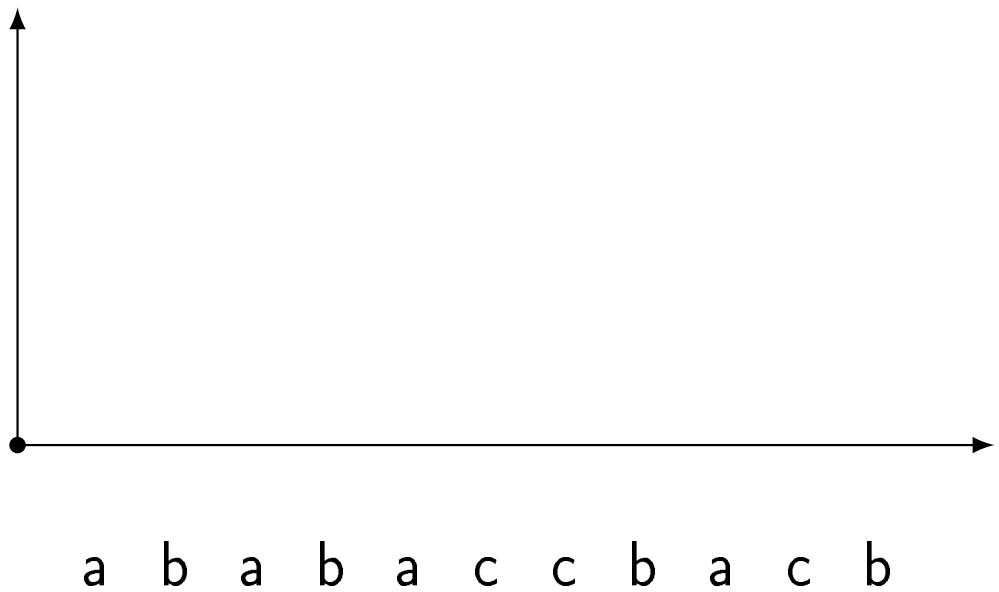

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Conclusion

For $k \geq 4$ and large n most logs end with word of length greater than M.

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α^5	$\{1, -3, -7, -11, \ldots\}$	0.535	2

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Thank you for your attention.